Final Exam- Review 2 - Answers

Peyam Ryan Tabrizian

Wednesday, December 9th, 2011

1)

$$\mathbf{x}(t) = Ae^{t}\begin{bmatrix}1\\1\\1\end{bmatrix} + Be^{2t}\begin{bmatrix}1\\2\\4\end{bmatrix} + Ce^{4t}\begin{bmatrix}1\\4\\16\end{bmatrix}$$

2)

$$\mathbf{x}(t) = Ae^{t}\begin{bmatrix}0\\0\\1\end{bmatrix} + B\left(e^{-t}\cos(4t)\begin{bmatrix}0\\2\\0\end{bmatrix} - e^{-t}\sin(4t)\begin{bmatrix}-1\\0\\0\end{bmatrix}\right) + C\left(e^{-t}\sin(4t)\begin{bmatrix}0\\2\\0\end{bmatrix} + e^{-t}\cos(4t)\begin{bmatrix}-1\\0\\0\end{bmatrix}\right)$$

3) Use 'generalized' eigenvectors:

$$\mathbf{x}(t) = Ae^{2t}\begin{bmatrix}1\\1\end{bmatrix} + B\left(te^{2t}\begin{bmatrix}1\\1\end{bmatrix} + e^{2t}\begin{bmatrix}\frac{1}{3}\\0\end{bmatrix}\right)$$

4) Proper frequencies: $\pm i, \pm\sqrt{3}i$

Proper modes:

$$\mathbf{v_1} = \begin{bmatrix}\sin\left(\frac{\pi}{3}\right)\\ \sin\left(\frac{2\pi}{3}\right)\end{bmatrix} = \begin{bmatrix}\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2}\end{bmatrix}$$

$$\mathbf{v_2} = \begin{bmatrix}\sin\left(\frac{2\pi}{3}\right)\\ \sin\left(\frac{4\pi}{3}\right)\end{bmatrix} = \begin{bmatrix}\frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2}\end{bmatrix}$$

5)

$$\frac{\pi^2}{3} + \sum_{m=1}^{\infty} \frac{4(-1)^m}{m^2} \cos(mx)$$

6)

$$u(x,t) = \frac{\pi}{2} \sum_{m=1}^{\infty} \widetilde{A_m} e^{-4m^2 t} \cos(mx)$$

where:

$$\widetilde{A_m} = \begin{cases} 0 & \text{if } m \text{ is even} \\ \frac{-4}{\pi m^2} & \text{if } m \text{ is odd} \end{cases}$$

8)

$$u(x,t) = 3\cos(9\pi t)\sin(3\pi x) + \frac{5}{12\pi}\sin(12\pi t)\sin(4\pi x)$$

8) Linearly independent (first write down the definition of linear independence, then cancel out e^x and x, and use the Wronskian with $x = 1$ on the remaining functions $1, x, x^2$)

9) $(0, 2)$

10) (a) $y(t) = Ae^t + Bte^t + Ct^2e^t$

(b) $y_p(t) = At^3e^t$

11)

$$y(t) = A\cos(t) + B\sin(t) + (\sin(t) - \ln|\sec(t) + \tan(t)|)\cos(t) - \cos(t)\sin(t)$$

$$y(t) = A\cos(t) + B\sin(t) - \ln|\sec(t) + \tan(t)|\cos(t)$$